Standard Form of a Quadratic Function

$$f(x) = a(x-h)^2 + k$$

- Where (h,k) is the vertex
- Where a determines minimum or maximum

Writing a Quadratic in Standard Form

- Vacate the "c"
- Take the "b" value, cut it in ½ and square it. Then divide by "a"

$$\left(\frac{b}{2}\right)^2 \div \frac{a}{1}$$

- Add the result to both sides of the equation and simplify.
- Factor out the leading coefficient from each term
- Factor the remaining expression as a "perfect square trinomial"
- Subtract right side from left.

Sum of Quadratic Formulas

$$\frac{-b-\sqrt{b^2-4ac}}{2a} + \frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2b}{2a}$$

$$= \frac{-b}{a}$$

Product of Quadratic Formulas

$$\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{b^2-\left(b^2-4ac\right)}{4a^2}$$

$$= \frac{b^2-b^2+4ac}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$= \frac{c}{a}$$

Position Function (Free Fall)

$$s(t) = \frac{-1}{2}gt^2 + V_O t + S_O$$

g = gravity $= 9.8\,{}^{m}/_{s^2}$ or $9.8\,{}^{m}/_{s^2}$

V_0 = initial velocity

S_0 = initial height

t = time

$s(t)$ = height at time t

Discriminant

$$b^2 - 4ac$$

Axis of Symmetry

$$x = \frac{-b}{2a}$$

$$x = h$$

Completing the square

- Vacate the "c" value
- Take the "b" value, cut it in ½ and square it. Then divide by "a"

$$\left(\frac{b}{2}\right)^2 \div \frac{a}{1}$$

- Add the result to both sides of the equation and simplify.
- Factor out the a value as a GCF
- Factor the left side as a "perfect s quare trinomial"
- Take the $\sqrt{\ }$ of both sides.
- Solve for x

Writing Quadratic Equations Given Roots

- Write each root as an equation
- Solve each for 0
- Distribute
- Use completing the square to put in standard form

$$x = 1 \qquad 7 = x$$

$$x - 1 = 0 \qquad 0 = x - 7$$

$$(x-1)(x-7) = 0$$

$$x^2 - 8x + 7 = 0$$

$$\left(\frac{-8}{2}\right)^2 \div 1 = 16$$

$$x^2 - 8x + 16 = -7 + 16$$

$$(x-4)^2 = 9$$

$$f(x) = (x-4)^2 - 9$$

$$V = (4,-9)$$

Quadratic Transformations

$R_{x\ axis}$ = Negate a,k

$R_{y\ axis}$ = Negate h

R_{origin} = Negate a,h,k

$$T_{m,n} = f(x) = a(x-h-m)^2 + k + m$$

$$D_n = \frac{a}{n} \wedge n(h,k)$$

Derivation of the Quadratic Formula

$$ax^2 + bx + c$$

$$ax^2 + bx = -c$$

$$\left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$$

$$\frac{b^2}{4} \div a = \frac{b^2}{4} \div \frac{a}{1} = \frac{b^2}{4} \cdot \frac{1}{a} = \frac{b^2}{4a}$$

$$ax^2 + bx + \frac{b^2}{4a} = -c + \frac{b^2}{4a}$$

$$ax^2 + bx + \frac{b^2}{4a} = \frac{-c}{1} \cdot \frac{4a}{4a} + \frac{b^2}{4a}$$

$$ax^2 + bx + \frac{b^2}{4a} = -\frac{4ac}{4a} + \frac{b^2}{4a}$$

$$ax^2 + bx + \frac{b^2}{4a} = \frac{b^2-4ac}{4a}$$

$$\left(x\sqrt{a} + \frac{b}{2\sqrt{a}}\right)^2 = \frac{b^2-4ac}{4a}$$

$$\sqrt{\left(x\sqrt{a} + \frac{b}{2\sqrt{a}}\right)^2} = \sqrt{\frac{b^2-4ac}{4a}}$$

$$x\sqrt{a} + \frac{b}{2\sqrt{a}} = \pm\frac{\sqrt{b^2-4ac}}{2\sqrt{a}}$$

$$x\sqrt{a} = -\frac{b}{2\sqrt{a}} \pm \frac{\sqrt{b^2-4ac}}{2\sqrt{a}}$$

$$x\sqrt{a} = \frac{-b \pm \sqrt{b^2-4ac}}{2\sqrt{a}}$$

$$\frac{1}{\sqrt{a}}\left(x\sqrt{a}\right) = \frac{1}{\sqrt{a}}\left(\frac{-b \pm \sqrt{b^2-4ac}}{2\sqrt{a}}\right)$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Solving A Quadratic Linear System

Graphically

- Graph both equations
- Look for intersections, if any
- Write as an ordered pair

Algebraically

- Substitute one equation into the other
- Solve for one variable
- Use your solution(s) to solve for the other
- Write as an ordered pair

Imaginary Numbers

$i^0=1 \quad i^1=i \quad i^2=-1 \quad i^3=-i$

$i^4=1$… (Pattern repeats)

Example: $\sqrt{-117} = 3i\sqrt{13}$